ON THE USE OF SPIRAL PIEZOMETER TAP CALIBRATION EQUATIONS AS A TRANSFER STANDARD BETWEEN ABSOLUTE DISCHARGE MEASUREMENT SYSTEMS

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ABSTRACT

It is demonstrated that the code specified power law equation for the analysis of Winter-Kennedy pressure tap data is an inappropriate form for the comparison of two measurement systems, introducing a spurious non-linearity into their inter-relationship. A general linear equation in the square root of the pressure tap difference is introduced to overcome the problem, which allows both proportional bias and zero offset errors to be identified. The modified equation is applied in a comparison of an acoustic time-of flight system with an acoustic scintillation system, both installed in the intake of Unit 4 of the Lower Granite Dam on the Snake River, Washington State, in 2004. The form of the proposed equation allows consistent confidence limits to be assigned to the fitting parameters.

INTRODUCTION.

The use of Winter-Kennedy piezometer taps to provide a relative measurement of the flow discharge rates is a long established practice within the hydro-electric generation industry, such measurements being employed to derive relative efficiency values for determining optimum cam operating profiles. To determine absolute efficiencies, the piezometer tap differences must be calibrated by independent discharge measurements.

The relationship between the discharge flow rate Q and the pressure tap difference D is specified in the industry codes PTC18 and IEC 60041 by an equation of the form

$$Q = KD^n \qquad (1)$$

The values of the exponent “n” are restricted to the range 0.48 to 0.52 about the expected theoretical value of 0.5, and in addition its range of applicability is limited to one half of the maximum flow rate. This form of equation is ill suited as a calibration equation, as the dimensions of the coefficient K are specific to a particular data set. For the original application of index testing, the code specified equation is perfectly satisfactory. However where the Winter–Kennedy tap calibrations are used to compare two different absolute measurement systems, equation (1) can lead to highly misleading results. This can be critical when discharge measurement systems are to be evaluated to a high level of accuracy, as shown by the following example.

Let Q be the discharge rate as determined by a standard procedure, and Q_1 the discharge measured by the system it is required validate, and let the relationship between the Q and Q_1 be of the form

$$Q_1 = kQ + c \qquad (2)$$

i.e. the system to be evaluated is linear, but has both a fixed offset bias c and a proportional bias k. In cases where the two measurement systems cannot be run simultaneously, the Winter-Kennedy pressure tap readings are used as a transfer standard. The following example demonstrates the problems associated with the code specified equation (1)

Let the calibrated flow rate Q be represented by the Winter-Kennedy calibration equation (1)

$$Q = KD^n$$

and the system to be evaluated represented by the relationship

$$Q_1 = K_1 D^m \qquad (3)$$

Eliminating the Winter-Kennedy tap reading D from equations (1) and (3) yields a nonlinear relationship between the two flow rates,

$$Q_1 = K_1 \left(\frac{Q}{K} \right)^{m/n} \qquad (4)$$

a quite different result from the actual linear relationship of equation (2). The general power law fit of the code has transformed a zero offset into a non-linearity, a gross distortion of the true state of affairs, and in addition, the proportional and offset biases cannot be individually identified.

AN ALTERNATIVE FITTING PROCEDURE.

The problems illustrated above arise because the form of the equation used to represent the Winter-Kennedy response is not compatible with the relationship between the two discharge measurement systems. There is no physical justification for the general power law fit of equation (1), and it has the effect of forcing the flow relationship to pass through zero in an artificial manner. At the high Reynolds numbers of the of discharge rates encountered in practice, there is a high expectation that the mean flow distribution in the vicinity of the pressure taps is independent of the flow rate, resulting in a square root relationship with the pressure tap difference. However, this high Reynolds number flow pattern will not necessarily hold as the flow approaches zero. For example, if the intake has regions of separated flow at high Reynolds numbers, these will collapse as the flow approaches zero, changing the overall flow pattern.

It is proposed instead that a relationship of the form

$$Q = aD^{0.5} + b \qquad (5)$$

be applied to fit Winter-Kennedy piezometer tap difference data. The zero offset constant b accounts for changing flow regimes as the flow approaches zero.

The following example illustrates the application of equation (5) to the two linearly related discharge data sets of equation (2). The two discharge rates are now represented by the Winter-Kennedy fits

$$Q = aD^{0.5} + b$$

and

$$Q_1 = a_1 D^{0.5} + b_1 \qquad (6)$$

Eliminating D between (4) and (5) now gives the following linear relationship between Q and $Q_{1.}$

$$Q_1 = \frac{a_1}{a} Q + \left(b_1 - \frac{a_1 b}{a} \right) \qquad (7)$$

The slope and intercept of the linear relationship between the two discharge data sets Q and Q_1 of equation (2) are determined as

$$k = \frac{a_1}{a} \quad \text{and} \quad c = b_1 - \frac{a_1}{a} b .$$

The use of equation (5) to represent the Winter Kennedy pressure differences thus correctly reproduces the linear relationship between the two discharge data sets. Any significant non-linearity in the system to be evaluated would be manifest as systematic fitting errors, thus allowing non-linearity to be distinguished separately from both a zero offset and a proportional bias. This equation also has the added advantage that unambiguous confidence intervals can be assigned to both the slope and the offset, unlike the power law where the treatment of fitting errors is problematical.

A practical example.

The following example is drawn from tests designed to compare the performance of two different acoustic techniques for flow measurement in low head plants, a time of flight system and an acoustic scintillation system, both mounted in the intake of the unit. These tests were conducted on Unit 4 at the Lower Granite generating plant, both with and without fish screens in place, Wittinger (1). Of interest here is the test series with the fish diversion screens in place, which introduce major irregularities into the flow distribution in the intake. The example shown here is from the on-cam runs. The acoustic scintillation data is presented here as the average of four repeat tests for each flow condition, the averaged blocks containing about 2 minutes of data, the code recommended length of averaging time. Figures 1 and 2 show the Winter- Kennedy plots in the linear form of equation (5).

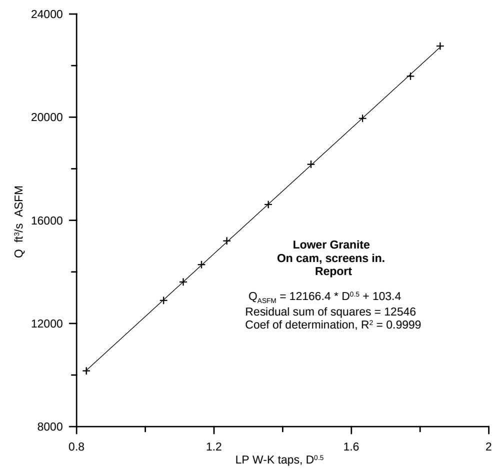

Figure 1. Winter – Kennedy plots for the scintillation system, after equation (5).

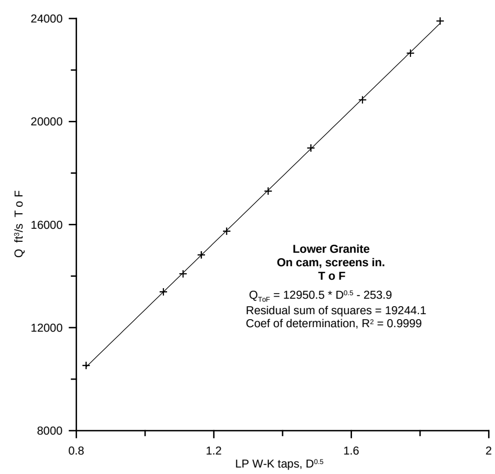

Figure 2. Winter – Kennedy plots for the time of flight system, after equation (5).

Both sets exhibit a high degree of correlation with the linear fits, as indicated by the coefficient of determination, R^2.

Applying the coefficients of the linear fits to equation (7) generates the following relationship between the outputs of the two flow systems.

$$Q_{T\,o\,F} = 1.064 Q_{ASFM} - 364 \qquad (8)$$

where $Q_{T\,o\,F}$ and Q_{ASFM} are the discharges measured by the time of flight and acoustic scintillation methods.

A direct plot the two discharges yields

$$Q_{T\,o\,F} = 1.064 Q_{ASFM} - 363, \quad \text{with } R^2 = 0.99989 \qquad (9)$$

an almost identical result. The very close agreement confirms the high correlations of the linear fits of figures 1 and 2.

Contrast this with the result of applying the simple power law to make the comparison. Logarithmic plots yield the following equations

$$Q_{T\,o\,F} = 12706 D^{0.5066}$$

and

$$Q_{ASFM} = 12266 D^{0.4971}$$

Combining these as in equation (4) yields

$$Q_{T\,o\,F} = 0.8047 (Q_{ASFM})^{1.026} \qquad (10)$$

a result clearly at odds with the linear relationships of equations (8) and (9).

Including the 95% confidence limits, the Winter-Kennedy fits from equation (5) are

$$Q_{ASFM} = (12166.4 \pm 29) D^{0.5} + (103.4 \pm 30)$$

$$Q_{ToF} = (12950.5 \pm 36) D^{0.5} - (254 \pm 37)$$

The constant term is significant in both cases, a consequence of the strong disturbances introduced into the flow by the fish diversion screens.

FURTHER EXAMPLES OF WINTER KENNEDY FITS.

Table 1 compares the results of the application of the linear form of equation (5) with the code recommended power law equation to sets of current measurements of flow in low head hydro-electric generating plants (2). The equations are compared on the basis of the standard deviation of the differences between the fits and the data, the fitting errors for the two equations being almost identical. In the case of the linear form of equation, unambiguous confidence intervals can be assigned to both slope and intercept, unlike the power law form where their definition is problematical. The table lists the 95% confidence intervals as percentages of the slope and intercept, and in most cases the intercept is not statistically significant.

The confidence intervals on the slopes range between about 1 and 2.5%, similar to the level of accuracy attributed to the original measurements. In the cases where the intercepts are statistically significant, the current meter measurements indicate that the intakes have uneven flow distributions.

Table 1. Comparison of Winter-Kennedy fitting equations.

			Linear fit					Power Law fit		
Plant	No.of points	Unit	Slope	95% Confd.	Intercept	95% Confd.	Fit error Std.dev.	Coefficient	Index	Fit error Std.dev.
Rock Island PH1	7	1 (L. taps)	6536	± 2.6%	-213.4	± 37%	15.2	6324	0.5204	16.7
	7	1 (H. taps)	5738	± 2.9%	-215	± 37%	15.3	5521	0.5206	16.9
	17	2	6821	±1.2 %	-32.9	± 170%	19.8	6788	0.5029	19.8
	16	3	7413	±3.3 %	59.5	± 161%	28.5	7365	0.5078	28.7
	7	4 (L. taps)	7024	±4.6 %	-183	±66 %	23.4	6850	0.5185	24.2
	7	4 (H. taps)	5571	±7.0 %	-155.0	±95 %	28.4	5415	0.5154	28.8
	14	6	4985	±1.45%	-144.7	±52%	42.9	4833	0.5114	39.3
Rock Island PH2	11	1	6270	±0.5%	-267.2	±41%	36.9	6008	0.5136	38.1
	16	6	6440	±2.6 %	116.3	± 470%	68.9	6577	0.4919	56.9
	9	7	6439	±1.9 %	116.3	± 313%	122.5	6577	0.4919	123.3
	16	8	6306	±1.5%	29.9	±970%	122.6	6354	0.4948	126.0
Rocky Reach	9	2	8538	±1.1%	139.2	±120%	49.0	8694	0.4916	58.6
	9 Ott	5	8776	±1.14 %	42.8	±309 %	49.7	8827	0.4967	52.3
	9 Neyrpic	5	8849	±1.14%	53.3	±372%	67.6	8924	0.4949	76.4
Dalles	11	5	7694	±3.5%	27.0	±1600%	127.0	7719	0.4989	127.0
	14	8	7915	±1.34 %	-44.3	±432 %	69.0	7881	0.5005	71.0
	6	9	7744	± 0.90%	-92.8	±90 %	22.0	7648	0.5052	24.3
	10	10	7753	±1.8%	56.8	±430%	74.0	7816	0.4962	75.4
Bonneville	7	2	7773	±2.'7%	-26.0	±715%	49.9	7746	0.5014	50.3
	7	6	7738	±3.5 %	213.6	±132 %	106.9	7980	0.4829	106.5
	7	7	7729	±1.1%	170.4	±52%	22.6	7909	0.4893	19.3
McNary	11	1	7954	±1.4%	-234.0	±60%	48.0	7711	0.5128	46.0
	8	2	8100	±0.8 %	-9.5	±775 %	24.2	8090	0.5002	24.3
	13	6	8250	±1.0 %	-56.7	±207 %	46.3	8198	0.5000	52.8
	13	10	8098	±1.3%	-68.7	±197%	60.0	8032	0.5014	59.0
Ice Harbor	14	1	8352	±1.60%	-247.0	±80%	70.0	8115	0.5096	77.0

CONCLUSIONS.

The code specified equation for the interpretation of Winter-Kennedy pressure tap data cannot be used for the inter-comparison of different measurement systems, as the power law form introduces a spurious non-linearity into the comparison. The use of the linear form of equation

$$Q = aD^{0.5} + b$$

removes the spurious non-linearity, and allows for the identification of the two independent forms of bias, a proportional bias and a constant offset. In addition, unambiguous confidence limits can be assigned to the fit.

The application of both the suggested linear form of equation and the code recommended power law to sets of current meter flow data shows that the differences in the quality of the fits of the two equations are negligible, and that for most of the examples shown the constant term of the linear form is not statistically significant at the 95% confidence level.

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